

SOLUTION KEY

The University of Chicago
Department of Economics

ECON 20100: Elements of Economic Analysis II

Final Examination

Thursday, December 11, 2025
12:30 PM - 14:30 PM

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Name:

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Instructions

- This is a **closed-book, closed-notes** examination.
- You have **120 minutes** to complete this exam.
- The exam consists of **3 problems** worth a total of **100 points**.
- Write your answers in the spaces provided. If you need additional space, use the back of the page and clearly indicate the continuation.
- Show all your work to receive full credit. Partial credit will be given for correct approaches even if the final answer is incorrect.
- Calculators are **not** permitted. Leave answers in simplified form where appropriate.
- Write legibly. Illegible answers will not receive credit.

Problem 1**[45 points]**

Bobus is producing airplanes using capital K and labor L . It is operating under a Cobb-Douglas technology:

$$f(K, L) = \frac{1}{10} K^{\frac{2}{3}} L^{\frac{1}{3}}.$$

It hires labor at a wage w and leases capital at a rental rate r . Currently, the input prices are $w = \frac{1}{3}$ and $r = \frac{2}{3}$, so the input price ratio is $\frac{w}{r} = \frac{1}{2}$.

(a) Show that the firm's total cost function is $c(q) = 10q$.

[6 pts]**Solution:**

To find the cost function, we need to solve the cost minimization problem:

$$\begin{aligned} \min_{K, L} \quad & rK + wL \\ \text{s.t.} \quad & \frac{1}{10} K^{\frac{2}{3}} L^{\frac{1}{3}} = \bar{q} \end{aligned}$$

The optimality condition for the firm:

$$2 \frac{L}{K} = \frac{r}{w} \Rightarrow K = 2L \frac{w}{r}.$$

Plugging that into the technology constraint to find the conditional factor demands:

$$\frac{1}{10} 2^{\frac{2}{3}} L^{\frac{2}{3}} \left(\frac{w}{r} \right)^{\frac{2}{3}} L^{\frac{1}{3}} = \bar{q} \Rightarrow L = 10 \times 2^{-\frac{2}{3}} \frac{\bar{q}}{\left(\frac{w}{r} \right)^{\frac{2}{3}}}.$$

$$K = 10 \times 2^{\frac{1}{3}} \left(\frac{w}{r} \right)^{\frac{1}{3}} \bar{q}.$$

Plugging the conditional factor demands into the objective to get the total cost function:

$$w \times 10 \times 2^{-\frac{2}{3}} \frac{\bar{q}}{\left(\frac{w}{r} \right)^{\frac{2}{3}}} + r \times 10 \times 2^{\frac{1}{3}} \left(\frac{w}{r} \right)^{\frac{1}{3}} \bar{q} = 10 \times \left[w^{\frac{1}{3}} r^{\frac{2}{3}} 2^{-\frac{2}{3}} + w^{\frac{1}{3}} r^{\frac{2}{3}} 2^{\frac{1}{3}} \right] \bar{q} = 10\bar{q}.$$

At the moment, Bobus is the monopolist on the market for private jets, and it offers only one type of plane for a per-unit price p . The inverse demand for private jets is given by $p = 90 - 2q$.

(b) Set up the monopolist's profit maximization problem in this setting.

[4 pts]**Solution:**

$$\begin{aligned} \max_{p, q} \quad & pq - 10q \\ \text{s.t.} \quad & p = 90 - 2q \end{aligned}$$

(c) Show that the monopolist's profit-maximizing price and quantity are $p^* = 50$ and $q^* = 20$. [4 pts]

Solution:

$$90 - 2q - 2q = 10 \Rightarrow 4q = 80 \Rightarrow q^* = 20$$

$$p^* = 90 - 2 * 20 = 50$$

The marketing analysts of Bobus have found that there are two clear segments of the market, defined by the country of origin of the consumer.

Consumers from one set of countries have the following inverse demand function:

$$p_1 = 50 - 4q_1,$$

while the consumers from the remaining set of countries have the following inverse demand function:

$$p_2 = 150 - q_2.$$

The management of Bobus is approaching you to help them predict the impact of moving to an alternative plane distribution scheme that takes advantage of this market segmentation. It is considering to set different per-unit prices for the two segments, while it can verify with certainty the country of origin of the consumer.

(d) Set up the profit maximization problem for Bobus in this scenario. [5 pts]

Solution:

$$\begin{aligned} \max_{p_1, p_2, q_1, q_2} \quad & p_1 q_1 + p_2 q_2 - 10(q_1 + q_2) \\ \text{s.t.} \quad & p_1 = 50 - 4q_1 \\ & p_2 = 150 - q_2 \end{aligned}$$

(e) Show that the optimal prices for the two segments are $p_1^* = 30$ and $p_2^* = 80$. [5 pts]

Solution:

$$50 - 4q_1 - 4q_1 = 10 \Rightarrow q_1^* = 5 \Rightarrow p_1^* = 30$$

$$150 - q_2 - q_2 = 10 \Rightarrow q_2^* = 70 \Rightarrow p_2^* = 80$$

The management of Bobus also learns that there is another company, Airing, that is considering to enter the same market that Bobus is operating in. The total cost function of Airing is $c(q) = 30q$.

Understanding that if Airing does enter the market Bobus will maintain a dominant position, the management of Bobus is asking you to help them predict the impact of Airing's entry on their profits.

You can interpret the dominant position as the ability to move first in this strategic interaction. Assume that neither of the companies is doing any kind of price discrimination.

(f) Set up the sequential game that captures the strategic interaction between Bobus and Airing, in case Airing decides to enter the market. [5 pts]

Solution:

Players: Bobus, Airing

Histories: $\{(q_1)\}$, $\{(q_1, q_2)\}$, where $q_1 \in [0, \infty)$, $q_2 \in [0, \infty)$

Player function: $P(\emptyset) = 1$, $P(q_1) = 2$.

Payoff functions: $\pi_1(q_1, q_2) = pq_1 - 10q_1$, $\pi_2(q_1, q_2) = pq_2 - 30q_2$.

(g) Set up the profit maximization problem for both firms in this setting.

[5 pts]

Solution:

Airing's problem:

$$\begin{aligned} \max_{q_2} \quad & pq_2 - 30q_2 \\ \text{s.t.} \quad & p = 90 - 2(q_1 + q_2). \end{aligned}$$

Bobus's problem:

$$\begin{aligned} \max_{q_1} \quad & pq_1 - 10q_1 \\ \text{s.t.} \quad & p = 90 - 2(q_1 + q_2) \\ & q_2 = 15 - \frac{1}{2}q_1 \end{aligned}$$

(h) Characterize the Sub-Game Perfect Nash Equilibrium of this game.

[6 pts]

Solution:

Let's find the best response of Airing to any previous history:

$$90 - 2q_1 - 2q_2 - 2q_2 = 30 \Rightarrow q_2 = \frac{1}{4}[60 - 2q_1] = 15 - \frac{1}{2}q_1$$

Let's solve for the optimal action of Bobus:

$$\begin{aligned} 90 - 2\left(q_1 + 15 - \frac{1}{2}q_1\right) - q_1 &= 10 \Rightarrow \\ 90 - 2q_1 - 30 &= 10 \Rightarrow \\ 2q_1 &= 50 \Rightarrow \\ q_1 &= 25 \end{aligned}$$

Therefore, the equilibrium is:

$$\left(q_1^* = 25, q_2^*(q_1) = 15 - \frac{1}{2}q_1\right).$$

In the equilibrium history, the outcomes are:

$$\begin{aligned} q_2 &= 15 - 12.5 = 2.5 \\ p^* &= 90 - 2 \times 27.5 = 35 \end{aligned}$$

(m) Show that Bobus's profits are reduced by 175 due to Airing's entry, compared to the status quo (analyzed in the beginning of this problem)? [5 pts]

Solution:

The formula for finding the answer:

$$(50 \times 20 - 10 \times 20) - (35 \times 25 - 10 \times 25) = -175.$$

Problem 2**[45 points]**

Consider an economy that consists of a (representative) consumer and two firms $i \in \{1, 2\}$ that each produces its own good.

The consumer chooses how much of each good to consume and how to allocate their time endowment T between leisure ℓ and working for the two firms. Their utility function is:

$$U(C_1, C_2, \ell) = C_1^{\frac{1}{3}} C_2^{\frac{1}{3}} \ell^{\frac{1}{3}},$$

where C_1 and C_2 are the quantities of goods 1 and 2 consumed, respectively.

Firm 1 operates under the following production function:

$$f_1(L_1) = L_1^{\frac{1}{2}},$$

where L_1 is the amount of labor hired by firm 1.

The production of good 1 entails heavy emissions v that hurt the production of good 2. The emissions depend one-to-one on the level of production of good 1:

$$v = q_1.$$

Firm 2 operates under the following production function, which reflects the negative impact of firm 1's emissions:

$$f_2(L_2; v) = L_2^{\frac{1}{2}} + (a - v^2), \quad v \leq \sqrt{a}, \quad a > 0,$$

where L_2 is the amount of labor hired by firm 2 and a is the maximum possible amount of pollution. For the rest of the problem you can assume that a is high enough to ensure that in any type of allocation v is less than $\sqrt{a}$.

(a) Set up the profit maximization problem for each firm in case firm 1 does not account for the effect of its production on v . **[5 pts]**

Solution:

Firm 1's problem:

$$\begin{aligned} \max_{q_1, L_1} \quad & p_1 q_1 - w L_1 \\ \text{s.t.} \quad & q_1 = \sqrt{L_1} \end{aligned}$$

Firm 2's problem:

$$\begin{aligned} \max_{q_2, L_2} \quad & p_2 q_2 - w L_2 \\ \text{s.t.} \quad & q_2 = \sqrt{L_2} + (a - v^2) \end{aligned}$$

(b) Show that in equilibrium, the levels of labor employed by firms 1 and 2 have to satisfy: [6 pts]

$$\frac{p_1}{p_2} = \frac{\sqrt{L_1}}{\sqrt{L_2}}.$$

Solution:

First-order conditions for both firms:

$$\frac{1}{2} \frac{1}{\sqrt{L_1}} = \frac{w}{p_1}$$

$$\frac{1}{2} \frac{1}{\sqrt{L_2}} = \frac{w}{p_2}$$

Dividing the second by the first:

$$\frac{p_1}{p_2} = \frac{\sqrt{L_1}}{\sqrt{L_2}}.$$

You can take as given that one of the optimality conditions for the consumer is

$$\frac{C_2}{C_1} = \frac{p_1}{p_2}.$$

(c) Combining the conditions derived so far, show that in equilibrium, the labor employed by firm 1 and firm 2 are related in the following way: [6 pts]

$$L_1^* = \frac{L_2^* + a\sqrt{L_2^*}}{1 + \sqrt{L_2^*}}.$$

Solution:

Combining the consumer and firm sides gives:

$$\frac{C_2}{C_1} = \frac{p_1}{p_2} = \frac{\sqrt{L_1}}{\sqrt{L_2}}.$$

Also using the market clearing conditions:

$$\begin{aligned} C_1^* &= \sqrt{L_1^*} \\ C_2^* &= \sqrt{L_2^*} + (a - v^{*2}). \end{aligned}$$

Bringing everything together:

$$\begin{aligned} \frac{C_2}{C_1} &= \frac{\sqrt{L_1}}{\sqrt{L_2}} \Rightarrow \\ \frac{\sqrt{L_2} + a - v^2}{\sqrt{L_1}} &= \frac{\sqrt{L_1}}{\sqrt{L_2}} \Rightarrow \\ L_2 + \sqrt{L_2}(a - v^2) &= L_1 \Rightarrow \\ L_2 + \sqrt{L_2}(a - L_1) &= L_1 \Rightarrow \\ L_2 + a\sqrt{L_2} - \sqrt{L_2}L_1 &= L_1 \Rightarrow \\ L_1 &= \frac{L_2 + a\sqrt{L_2}}{1 + \sqrt{L_2}} \end{aligned}$$

(d) Set up the social planner's problem in this setting.

[5 pts]

Solution:

$$\begin{aligned} \max_{C_1, C_2, \ell, L_1, L_2, v} \quad & C_1^{\frac{1}{3}} C_2^{\frac{1}{3}} \ell^{\frac{1}{3}} \\ \text{s.t.} \quad & C_1 = \sqrt{L_1} \\ & C_2 = \sqrt{L_2} + (a - v^2) \\ & v = C_1 \\ & L_1 + L_2 + \ell = T \end{aligned}$$

(e) Combine the first-order conditions in the social planner's problem to show that in the efficient allocation, the labor employed by firm 1 and firm 2 are related in the following way: [6 pts]

$$L_1^{\text{SP}} = \frac{L_2^{\text{SP}} + a\sqrt{L_2^{\text{SP}}}}{1 + 3\sqrt{L_2^{\text{SP}}}}.$$

Solution:

FOCs:

$$\frac{1}{3}C_1^{-\frac{2}{3}}C_2^{\frac{1}{3}}\ell^{\frac{1}{3}} = -\lambda_1 + \lambda_3 \quad (1)$$

$$\frac{1}{3}C_1^{\frac{1}{3}}C_2^{-\frac{2}{3}}\ell^{\frac{1}{3}} = -\lambda_2 \quad (2)$$

$$\frac{1}{3}C_1^{\frac{1}{3}}C_2^{\frac{1}{3}}\ell^{-\frac{2}{3}} = -\lambda_4 \quad (3)$$

$$\frac{1}{2}\frac{1}{\sqrt{L_1}}\lambda_1 = \lambda_4 \quad (4)$$

$$\frac{1}{2}\frac{1}{\sqrt{L_2}}\lambda_2 = \lambda_4 \quad (5)$$

$$-2v\lambda_2 = \lambda_3 \quad (6)$$

From Equation 6:

$$\frac{\lambda_3}{\lambda_2} = -2v. \quad (7)$$

Dividing Equation 4 by Equation 5:

$$\frac{\lambda_1}{\lambda_2} = \frac{\sqrt{L_1}}{\sqrt{L_2}}. \quad (8)$$

Dividing Equation 1 by λ_2 and using Equation 2, Equation 7 and Equation 8:

$$-\frac{\frac{1}{3}C_1^{-\frac{2}{3}}C_2^{\frac{1}{3}}\ell^{\frac{1}{3}}}{\lambda_2} = \frac{\lambda_1}{\lambda_2} - \frac{\lambda_3}{\lambda_2} \Rightarrow \frac{C_2}{C_1} + \frac{\sqrt{L_1}}{\sqrt{L_2}} = 2\sqrt{L_1} \quad (9)$$

$$\Rightarrow \frac{\sqrt{L_2} + a - L_1}{\sqrt{L_2}} + \frac{\sqrt{L_1}}{\sqrt{L_2}} = 2\sqrt{L_1} \quad (10)$$

$$\Rightarrow \frac{L_2 + a\sqrt{L_2} - L_1\sqrt{L_2}}{L_1} - 1 = 2\sqrt{L_2} \quad (11)$$

$$\Rightarrow L_2 + a\sqrt{L_2} - L_1\sqrt{L_2} - L_1 = 2\sqrt{L_2}L_1 \quad (12)$$

$$\Rightarrow L_2 + a\sqrt{L_2} = L_1\sqrt{L_2} + L_1 + 2\sqrt{L_2}L_1 \quad (13)$$

$$\Rightarrow L_1 = \frac{L_2 + a\sqrt{L_2}}{1 + 3\sqrt{L_2}} \quad (14)$$

(f) Is the efficient level of pollution v^{SP} higher or lower than the equilibrium one, v^* ? [5 pts]

Solution:

Since we have shown that $\frac{L_1^*}{L_2^*} > \frac{L_1^{\text{SP}}}{L_2^{\text{SP}}}$, $v^{\text{SP}} < v^*$.

Now, the legal institutions have developed to the extent that firm 2 has verifiable property rights over the state of the environment, which can be enforced in court. In this case, if firm 1's production pollutes the environment, it needs to compensate firm 2 at rate p_v .

(g) Set up the profit maximization problem for firm 1 and firm 2 in this setting. [6 pts]

Solution:

Firm 1's problem:

$$\begin{aligned} \max_{q_1, L_1, v} \quad & p_1 q_1 - w L_1 - p_v v \\ \text{s.t.} \quad & q_1 = \sqrt{L_1} \\ & v = q_1 \end{aligned}$$

Firm 2's problem:

$$\begin{aligned} \max_{q_2, L_2, v} \quad & p_2 q_2 - w L_2 + p_v v \\ \text{s.t.} \quad & q_2 = \sqrt{L_2} + (a - v^2) \end{aligned}$$

(h) Combining first-order conditions for the firms and the consumer, show that in equilibrium, [6 pts] the labor employed by firm 1 and firm 2 are related in the following way:

$$L_1^C = \frac{L_2^C + a\sqrt{L_2^C}}{1 + 3\sqrt{L_2^C}}.$$

Solution:

FOCs for firm 1:

$$p_1 = -\lambda_1 + \lambda_2 \quad (15)$$

$$w = -\lambda_1 \frac{1}{2} \frac{1}{\sqrt{L_1}} \quad (16)$$

$$p_v = \lambda_2 \quad (17)$$

FOCs for firm 2:

$$p_2 = -\mu \quad (18)$$

$$w = -\mu \frac{1}{2} \frac{1}{\sqrt{L_2}} \quad (19)$$

$$p_v = -2\mu v \quad (20)$$

Plugging Equation 17 into Equation 15 and dividing the result by Equation 16:

$$\frac{p_1 - p_v}{w} = 2\sqrt{L_1}. \quad (21)$$

Plugging Equation 18 into Equation 19:

$$\frac{w}{p_2} = \frac{1}{2} \frac{1}{\sqrt{L_2}}. \quad (22)$$

Multiplying Equation 21 and Equation 22:

$$\frac{p_1 - p_v}{p_2} = \frac{\sqrt{L_1}}{\sqrt{L_2}}. \quad (23)$$

Dividing Equation 20 by Equation 18:

$$\frac{p_v}{p_2} = 2v. \quad (24)$$

Bringing it all together (adding the consumer optimality condition):

$$\frac{C_2}{C_1} - 2v = \frac{\sqrt{L_1}}{\sqrt{L_2}}, \quad (25)$$

which is the same equation as in the social planner's problem.

Problem 3

For each of the following statements, argue if they are true, false, or uncertain.

- (a) In a simultaneous 2-by-2 game, if a profile of actions is Pareto-efficient, then it Pareto- [5 pts]
dominates every other profile of actions.

Solution:

False. One counterexample:

		Player 2	
		A	B
Player 1	A	0, 0	3, 1
	B	1, 3	2, 2

(B, B) is Pareto-efficient but does not Pareto-dominate (A, B) .

- (b) If a monopolist can distinguish between types of consumers (according to their willingness [5 pts]
to pay), it will extract the entire consumer surplus from each type of consumer.

Hint: Use graphs to support your argument.

Solution:

False, since the statement does not specify which pricing schemes are available to the monopolist. If the monopolist can only charge per-unit prices, then it won't extract all of the consumer surplus.

Scratch Paper

This page is for rough work only. Work on this page will not be graded.

– END OF EXAM –

Please review your answers before submitting.