

Problem Set 1

Due Friday, January 23rd at 6pm

Problem 1

For each of the following statements, argue if it is TRUE, FALSE, or UNCERTAIN.

1. If a production function $f(K, L)$ displays constant returns to scale, then $f_{KL} \equiv \frac{\partial^2 f(K, L)}{\partial K \partial L}$ must be strictly positive.
2. After an observed increase in the relative cost of labor, a firm hasn't changed its composition of inputs. We can conclude that this firm's technology is of the Leontieff type.
3. For a firm with any technology, if its input bundle is not cost-minimizing, the firm has to change the levels of all of its inputs to reduce the production cost.
4. The minimum point of ATC curve is right above the minimum point of AVC curve (at the same level of production q).

Problem 2

Consider a firm whose production function is $f(L, K) = L^{\frac{1}{2}} + K^{\frac{1}{2}}$.

1. Calculate the marginal products of labor and capital. Does the production function satisfy diminishing marginal product for each input?
2. What is the MRTS for this production function?
3. Now, let's examine if this production function features diminishing MRTS.
 1. First, consider an isoquant for this production function corresponding to the level of production $\bar{q} = 3$. Calculate levels of K for $L = 1$ and $L = 4$ necessary to stay on this isoquant.
 2. Using the formula for MRTS, compare its value across the two calculated production plans. Is the outcome consistent with diminishing MRTS?
 3. Show generally that this production function features diminishing MRTS.
4. Does this production function have increasing, constant, or decreasing returns to scale?

Problem 3

Consider a firm with the production function $f(L, K) = K \ln(L)$. For the rest of this problem, assume that we are in the short run, so the value of capital is fixed at $\bar{K}$.

1. Set up the cost minimization problem for this firm.
2. Derive the conditional factor demand for L .
3. Derive the total cost function.

4. Fixing $w = 2$, $r = 5$, and $\bar{K} = 2$, express total cost, variable cost, and fixed cost as functions of output q .
5. Sketch these functions on the same plot.
6. Fixing w , r , and $\bar{K}$ at the same values as before, express ATC, AVC, and AFC as functions of output q .
7. Sketch these functions on the same plot.
8. Derive the marginal cost function.
9. On a separate graph, draw the ATC, AVC, and MC. What do we know about the intersection of MC with each of the other two curves? Why is this the case (explain in words not math)?
10. Now, let's examine how the conditional factor demand and the total cost function respond to an increase in the fixed capital to $\bar{\bar{K}} = 4$.
 1. Sketch the conditional factor demand (as a function of q) under both values of K on the same plot. Describe the difference.
 2. Sketch the total cost function (as a function of q) under both values of K on the same plot. Describe the difference.
 3. Suggest an intuition for the observed patterns.

Problem 4

Consider a firm whose production function is $f(K, L) = K^{\frac{1}{2}} + L^{\frac{1}{2}}$.

1. Set up the cost minimization problem for this firm.
2. Derive the total cost function $c(q)$ for this firm.
3. Set up the indirect profit maximization problem for this firm.
4. Characterize the solution of this problem and derive profits as a function of the output price $\pi(p)$.
5. Now, set up the direct profit maximization for this firm, in which you jointly optimize over the output level q and the use of inputs K and L .
6. Using the Lagrangian method (in the same way as in cost minimization), derive again the profit function $\pi(p)$. Compare it with the profit function found above (in 4).

Problem 5

Consider a firm whose total cost function is $c(q) = 3q$. It can sell its output on a perfectly competitive market where the price has set at $p^* = 2$.

1. Knowing that the firm uses two types of inputs (say, K and L), can you hypothesize what production technology does this firm operate under?
2. Assume you have established (from some previous analysis) that this firm's production technology is of the Cobb-Douglas type. Can you pinpoint its *exact* form? (That is, can you pinpoint the values of its parameters?)

3. Set up the indirect profit maximization problem for this firm.
4. Characterize the solution of the profit maximization problem and explain it in words.
5. Now, the government is not content with this outcome, so it introduces a regressive subsidy policy $s(q)$ that is characterized by the following marginal schedule:

$$s(q) := \begin{cases} 3 - .6q & \text{if } q \leq 5 \\ 0 & \text{if } q > 5 \end{cases}$$

1. Describe this subsidy policy in words.
2. What is the effective total cost function $c^s(q)$ of the firm under this policy?
3. Plot $c^s(q)$ and the revenue function on the same graph.
4. Characterize the solution of the profit maximization problem under this subsidy policy.
5. (*Extra points.*) In general, the government might want to ensure that the firm produces some target level of output q^G . It is asking you to tell them how they should change the subsidy schedule depending on the target level q^G . Help the government by characterizing the dependence of $s(q)$ on the target q^G .