

Problem Set 2

Due Friday, January 30th at 6pm

Problem 1

For each of the following statements, argue if it is TRUE, FALSE, or UNCERTAIN.

1. In a perfectly competitive market, it is impossible to see a firm operating with negative profits.
2. If the number of firms on a perfectly competitive market is low enough, there could be supply shortage in the short run.
3. In the long run in a perfectly competitive market, all firms supplying positive output must have the same cost function.
4. In the short run in a perfectly competitive market, aggregate producer surplus (the sum of producer surpluses of all firms on the market) increases with the number of firms.

Problem 2

Consider a set of firms $i \in \{1, 2, 3\}$, whose cost functions are characterized by $c_i(q) = a_i q^2 + 10$, with $a_i > 0$.

1. What is the marginal cost function of firm i ?
2. What is the supply function of firm i ?
3. Derive the industry supply function for the industry consisting of these three firms.
4. Now, assume that the cost function parameters take the following values: $a_1 = 2$, $a_2 = 3$, $a_3 = 1$. Some analyst is suggesting to make the analysis of the market easier by defining a theoretical 'representative' firm, whose behavior in theory matches the behavior of the whole industry.
 - 4.a. This analyst's naive proposition for the representative cost function is $c_r(q) = \bar{a}q^2 + 10$, with $\bar{a} = 2 + 3 + 1 = 6$. Does this representative cost function achieve the original goal of describing the aggregate industry behavior?
 - 4.b. Can you propose a better way to derive the representative cost function?
 - 4.c. Given some market price p^* , calculate the sum of individual producer surpluses and the producer surplus of the representative firm constructed by you in the previous sub-question. Are they the same?

Problem 3

Consider a set of firms $i \in \{1, 2, 3, 4, 5\}$, whose average cost functions are characterized by $AC_i(q) = (q - a_i)^2 + 5$, with $a_i > 0$.

The inverse market demand for the good produced by these firms is characterized by $D^{-1}(q) = b - q$ for some $b > 0$. Entry to the market is free to the firms.

1. Assume that the cost function parameters take the following values: $a_1 = 1, a_2 = 2, a_3 = 5, a_4 = 6, a_5 = 6$. What are the minimum price levels $\bar{p}_i$ at which these firms would operate on a perfectly competitive market? For each firm i , what quantity $\bar{q}_i$ would it want to produce under $\bar{p}_i$?
2. Derive the marginal cost function for each firm i .
3. Characterize the individual supply functions of these firms.
4. What is the minimum level of b for which there will be any output sold on the market? Denote that level b_1 . Which firm(s) i is going to be operating in this case?
5. If the market demand shifts upwards so that b becomes $b_1 + 1$, how many firms are going to be operating on the market? Why?
6. What is the minimum increase in b starting from b_1 that will incentivize new firms to enter the market? Which firm(s) would enter the market first?
7. If we observe that all 5 firms are active on the market, what can we say about the value of b ? (What is its lowest possible level?)
8. (*Extra points*) Assume that the demand parameter is given to be $b = 10$ and there are 100 firms operating on the market. Can you come up with some set of values $\{a_i\}_{i=1}^{100}$ that would be consistent with this scenario? Combining this observation with your answer to the previous sub-question, discuss what is necessary for the profits to be close to zero in a competitive market.

Problem 4

The handmade snuffbox industry is composed of 100 identical firms, each having short-run total costs given by $STC(q) = 0.5q^2 + 10q + 5$, where q is the output of snuffboxes per day.

1. What is the short-run supply curve for each snuffbox maker? What is the short-run supply curve for the market as a whole?
2. Suppose the demand for total snuffbox production is given by $Q = 1100 - 50P$. What will be the equilibrium in this marketplace? What will each firm's total short-run profits be?
3. Graph the market equilibrium and compute total short-run producer surplus in this case.