

Problem Set 3

Due Friday, February 6th at 6pm

Problem 1

For each of the following statements, argue if it is TRUE, FALSE, or UNCERTAIN.

1. In case the consumer has a linear utility over consumption and leisure, if the real wage is low enough, the consumer does not work at all in general equilibrium.
2. In the Robinson Crusoe economy, if the firm becomes more productive (that is, the marginal product of labor increases for any level of labor input), then the ratio of consumption to leisure will increase in general equilibrium.

Problem 2

Consider a Robinson Crusoe Economy with Cobb-Douglas utility function $U(C, \ell) = \sqrt{C} + \sqrt{\ell}$, initial endowment of time $T = 24$, and production technology $q = f(L) = L$, which produces the consumption good from labor input.

1. Set up the profit maximization problem and derive the labor demand function.
2. Set up the consumer problem and derive the consumer demand and labor supply functions.
3. Find the equilibrium price and quantities.
4. Set up the social planner's problem and verify that its solution corresponds to the equilibrium levels of consumption and leisure.

Problem 3

Consider a Robinson Crusoe economy, where the consumer's utility over consumption C and leisure ℓ has a log-linear form:

$$U(C, \ell) = \log \ell + C.$$

Consumer's time endowment is $T \geq 1$.

The firm uses labor to produce the good and operates under the following technology:

$$f(L) = L^\beta, \quad 0 < \beta \leq 1.$$

Let's analyze the general equilibrium in this economy.

1. Set up the consumer's problem.
2. Characterize the consumer's labor supply function. Does it depend on the time endowment T ?
3. Set up the profit maximization problem for the firm.
4. Consider the case of linear technology (i.e., $\beta = 1$).

- 4.a. What is the labor demand function in this case? Can you eliminate values of w that cannot be consistent with the definition of the equilibrium?
- 4.b. Characterize the general equilibrium and plot it on the graph with the PPF and isoutility curves.
- 4.c. Now, consider what happens if the time endowment increases to $T' > T$. How does the equilibrium change?
5. Consider the case of $\beta = \frac{1}{2}$, where the marginal product is decreasing in labor use.
 - 5.a. Derive the labor demand function, characterize the general equilibrium, and plot it on the graph with the PPF and isoutility curves.
 - 5.b. Consider again what happens if the time endowment increases to T' . Does the equilibrium amount of labor L^* change? Discuss in light of your answers to sub-questions 2 and 4.3.

Problem 4

Consider an economy with goods 1 and 2, a consumer, and 2 firms producing goods 1 and 2, respectively.

The consumer's time endowment is $T \geq 1$, and they can allocate their time between leisure ℓ , working for firm 1, and working for firm 2. Their utility function over the 2 goods and leisure ℓ is

$$U(C_1, C_2, \ell) = \log \ell + C_1 C_2.$$

Both firms operate under the linear technology so that $f_1(L) = L$ and $f_2(L) = L$. For each firm i , it compensates the labor input at wage rate w_i .

Let's analyze the efficient allocation and the general equilibrium in this economy.

1. Set up the social planner's problem.
2. Find the feasible efficient allocation in this economy.
3. Now let's set up individual problems of the consumer and both firms, and define the equilibrium in this economy.
 - 3.a. Denote s_1 and s_2 as shares of the non-leisure time $T - \ell$ that the consumer spends on working for firm 1 and 2, respectively. Set up the consumer problem, in which they choose how much of each good to consume, how much time to spend on leisure, and how to allocate the remaining time between working for 2 firms.
 - 3.b. Set up the profit maximization problem for both firms.
 - 3.c. Following the structure of the general equilibrium definition in the Robinson Crusoe economy considered in class, define the general equilibrium in this economy. That is, list the variables that are determined in equilibrium and all of the conditions they need to satisfy in equilibrium.
 - 3.d. Solve the consumer's problem and characterize its demand function for both of the goods and its labor supply functions for both of the firms.

Hint: Start by characterizing optimal s_1^* and s_2^* , and then solve for the remaining choice variables. What makes the consumer prefer to work for either of the firms?

Write the first order conditions with respect to ℓ , C_1 , and C_2 . What's the relation between optimal choices of C_1 and C_2 ? You can postpone solving for the optimal value of the Lagrange multiplier until later.

- 3.e. Solve the profit maximization problem for both firms and characterize their labor demand functions.
- 3.f. Keeping in mind the general equilibrium definition, eliminate values of w_1 and w_2 that would not be consistent with it, just based on the form of the labor demand functions.

Hint: What do we know about firms with linear technologies? Keep in mind the types of scenarios we considered in class when solving the profit maximization problem in general. What are the firms' profits then in equilibrium?

- 3.g. Characterize the general equilibrium in this economy (the values should only depend on T).
 - 3.g.a. Start by showing that $w_1^* = w_2^*$ in the equilibrium. Rely on a proof by contradiction, i.e., argue why is it impossible for the wages to be different in general equilibrium?
 - 3.g.b. Normalize prices by setting $p_1^* = 1$.
 - 3.g.c. Combining all the observations from above, simplify the budget constraint of the consumer, and using the first-order conditions for ℓ , C_1 , and C_2 , solve for the optimal value of the Lagrange multiplier.