

Cost curves and profit maximization

ECON 201 - Winter 2026

Week 2: January 14th

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Cost curves

Today's Agenda

1. Variable vs fixed costs (short run vs long run)
2. Properties of cost curves
3. Derivation of cost curves for the Cobb-Douglas technology

Recall

Cost Minimization Problem

$$\begin{array}{ll} \min_{z} & w \cdot z \\ \text{s.t.} & f(z) = \bar{q} \end{array}$$

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- v are the (variable) inputs that the firm can change in the short run;

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Split z into 2 parts: $z = (v, x)$, where

- v are the (variable) inputs that the firm can change in the short run;
- x are the inputs that the firm can only change in the long run

Short-run cost minimization

Short-run Cost Minimization Problem

$$\begin{aligned} \min_{z=(v,x)} \quad & \tilde{w} \cdot v + \bar{w} \cdot x \\ \text{s.t.} \quad & f(v, x) = \bar{q} \\ & x = \bar{x} \end{aligned}$$

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- $\bar{x}$ becomes an additional parameter of the problem alongside $w \equiv (\tilde{w}, \bar{w})$ and $\bar{q}$

Short-run cost functions

Short-run costs

$$c_s(w, q, \bar{x}) \equiv \underbrace{\tilde{w} \cdot v^*(w, q, \bar{x})}_{\text{variable costs}} + \underbrace{\bar{w} \cdot \bar{x}}_{\text{fixed costs}}$$

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Average variable costs

$$AVC(q) \equiv \frac{\tilde{w} \cdot v^*(w, q, \bar{x})}{q}$$

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Average variable costs

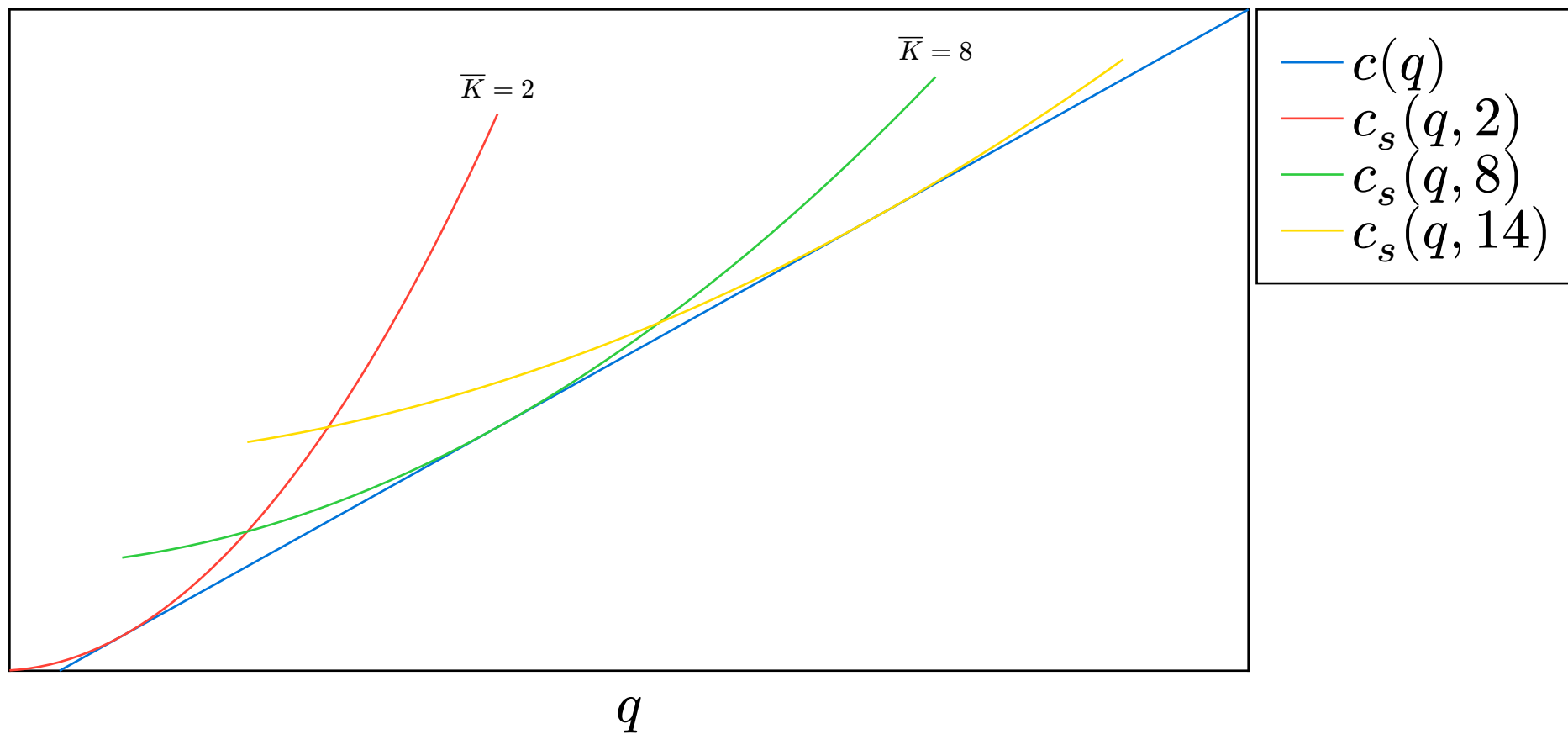
$$AVC(q) \equiv \frac{\tilde{w} \cdot v^*(w, q, \bar{x})}{q}$$

Average fixed costs

$$AFC(q) \equiv \frac{\bar{w} \cdot \bar{x}}{q}$$

Short run vs long run

Consider a Cobb-Douglas technology: $q = \bar{K}^{\frac{1}{2}} L^{\frac{1}{2}}$:



Profit maximization

Recall

A firm chooses the level of output q and the composition of inputs z to maximize its profits π :

$$\max_{q,z} \quad \pi \equiv pq - c$$

$$\text{s.t.} \quad p = p^*$$

$$q = f(z)$$

$$c = w^* \cdot z$$

- q is the output quantity
- z are the inputs (factors of production)
- f is the production function
- p^* is the market price for the produced good
- w^* are the market prices for the inputs

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$$\max_{q,z} \quad \pi \equiv pq - c$$

$$\text{s.t.} \quad p = p^* \quad \text{perfect competition}$$

$$q = f(z) \quad \text{technological constraint}$$

$$c = w^* \cdot z \quad \text{cost of production}$$

- q is the output quantity
- z are the inputs (factors of production)
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Connecting with cost minimization

Splitting Profit Maximization Problem

$$\max_q \max_z \quad \pi \equiv pq - c$$

$$\text{s.t.} \quad p = p^*$$

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Connecting with cost minimization

Splitting Profit Maximization Problem

$$\begin{aligned} \max_q \max_z \quad & \pi \equiv pq - c \\ \text{s.t.} \quad & p = p^* \\ & q = f(z) \\ & c = w^* \cdot z \end{aligned}$$

Inner problem solves for the optimal input mix z^* conditional on q .

Connecting with cost minimization

(Indirect) Profit Maximization Problem

$$\begin{aligned} \max_q \quad & \pi(q) \equiv pq - c \\ \text{s.t.} \quad & p = p^* \\ & c = c(q) \end{aligned}$$

Connecting with cost minimization

(Indirect) Profit Maximization Problem

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4 scenarios:

Connecting with cost minimization

(Indirect) Profit Maximization Problem

$$\begin{aligned} \max_q \quad & \pi(q) \equiv pq - c \\ \text{s.t.} \quad & p = p^* \\ & c = c(q) \end{aligned}$$

4 scenarios:

- $q^* = 0$
- $q^* \rightarrow \infty$
- Unique $0 < q^* < \infty$
- Any $q \in [0, \infty)$ is a solution

Profit maximization

$$q^* = 0$$

Interpretation: Firm does not enter the market

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Which is equivalent to:

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Intuitively:

- $c(q)$ grows too early and too fast

$$q^* \rightarrow \infty$$

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Which is equivalent to:

$$\frac{\partial c(q)}{\partial q} < p^* \quad \forall q \in [0, \infty)$$

Intuitively:

- Either $c(q)$ grows very slowly
- Or $p = p^*$ is a bad assumption

Unique $0 < q^* < \infty$

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Intuitively:

- $c(q)$ starts low and grows slowly for smaller values of q but accelerates later

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$$\frac{\partial \pi(q)}{\partial q} = 0 \quad \forall q \in [0, \infty)$$

Which is equivalent to:

$$\frac{\partial c(q)}{\partial q} = p^* \quad \forall q \in [0, \infty) \quad \text{and} \quad c(0) = 0$$

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Intuitively:

- $c(q)$ is linear $\Rightarrow$ marginal cost is constant
- No fixed costs