

General equilibrium with production

ECON 201 - Winter 2026

Week 4: January 28th

Sasha Petrov

This week's Agenda

1. Robinson Crusoe economy
2. Definition of equilibrium in Robinson Crusoe economy
3. Characterizing the equilibrium in Robinson Crusoe economy
4. Efficiency and social planner problem
5. First welfare theorem in Robinson Crusoe economy

Robinson Crusoe economy

Motivation

Recall that up until now:

- Market demand and labor supply were (largely) absent in the analysis



- p^* and w were determined outside of the framework

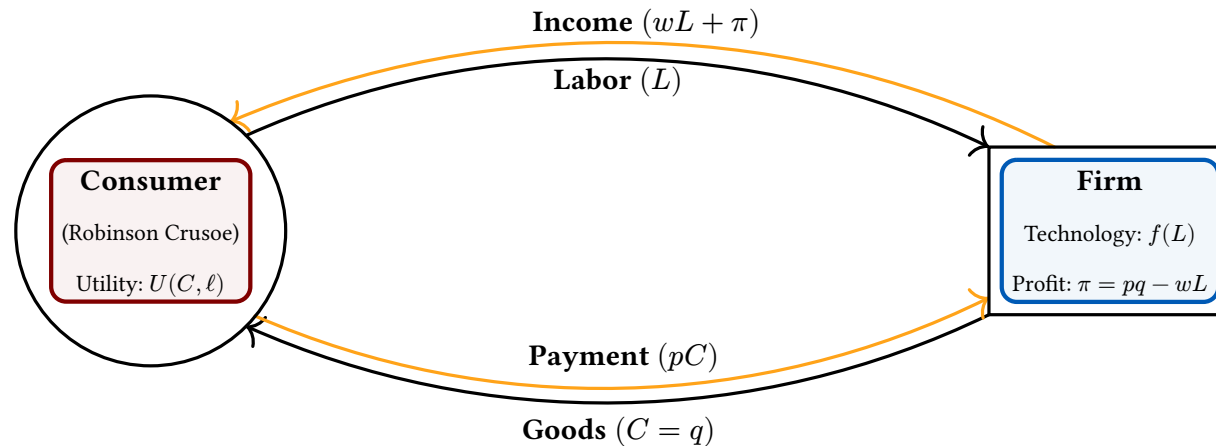
In **general equilibrium**, the determination of p^* and w becomes part of the framework.

Building blocks

- 2 goods in the economy: Consumption C and leisure ℓ
- Consumer (Robinson Crusoe):
 - Can give up leisure to work and earn wage w
 - Can spend w on the consumption good C
- Firm
 - Profit-maximizing price-taker w/ technology f
 - Hires labor L (Robinson Crusoe) and sells output q (to Robinson Crusoe)
 - Rebates profits π to Robinson Crusoe

An interpretation: Capture the allocation of resources, income flows, and spending decisions in the whole economy, using representative agents.

Building blocks



Real flows:

- Labor supply
- Goods provision

Money flows:

- Income (wages + profits)
- Consumption payment

Consumer's problem

Consumer

Given time endowment T , price of consumption p , wage level w , and rebated profits π , consumer is choosing consumption C and leisure ℓ to maximize the utility function U :

$$\begin{aligned} \max_{C, \ell} \quad & U(C, \ell) \\ \text{s.t.} \quad & pC \leq w(T - \ell) + \pi \end{aligned}$$

Firm's problem

Firm

Given price of consumption p , wage level w , and technology f , the firm is choosing input L and production q to maximize its profits:

$$\begin{aligned} \max_{q,L} \quad & \pi \equiv pq - wL \\ \text{s.t.} \quad & q = f(L) \end{aligned}$$

General Equilibrium

Equilibrium definition

An equilibrium in the Robinson Crusoe economy is a level of consumption C^* , a level of production q^* , a level of labor input L^* , a price p^* , a wage w^* , and profits π^* such that

- C^* and L^* solve the consumer's problem given p^* , w^* , and π^* ;
- q^* and L^* solve the firm's problem given p^* and w^* ;
- the output market and the labor market clear.

Market clearing

Output market

The demanded quantity of consumption is equal to the produced quantity:

$$C^* = q^*$$

Labor market

The supplied level of labor is equal to the firm's demand for labor:

$$\ell^* = T - L^*$$

Equilibrium conditions

RCE Equilibrium

An equilibrium in RCE is characterized by:

$$\frac{\text{MU}(C^*)}{\text{MU}(\ell^*)} = \frac{p^*}{w^*}$$

$$\frac{\partial f(L^*)}{\partial L} = \frac{w^*}{p^*}$$

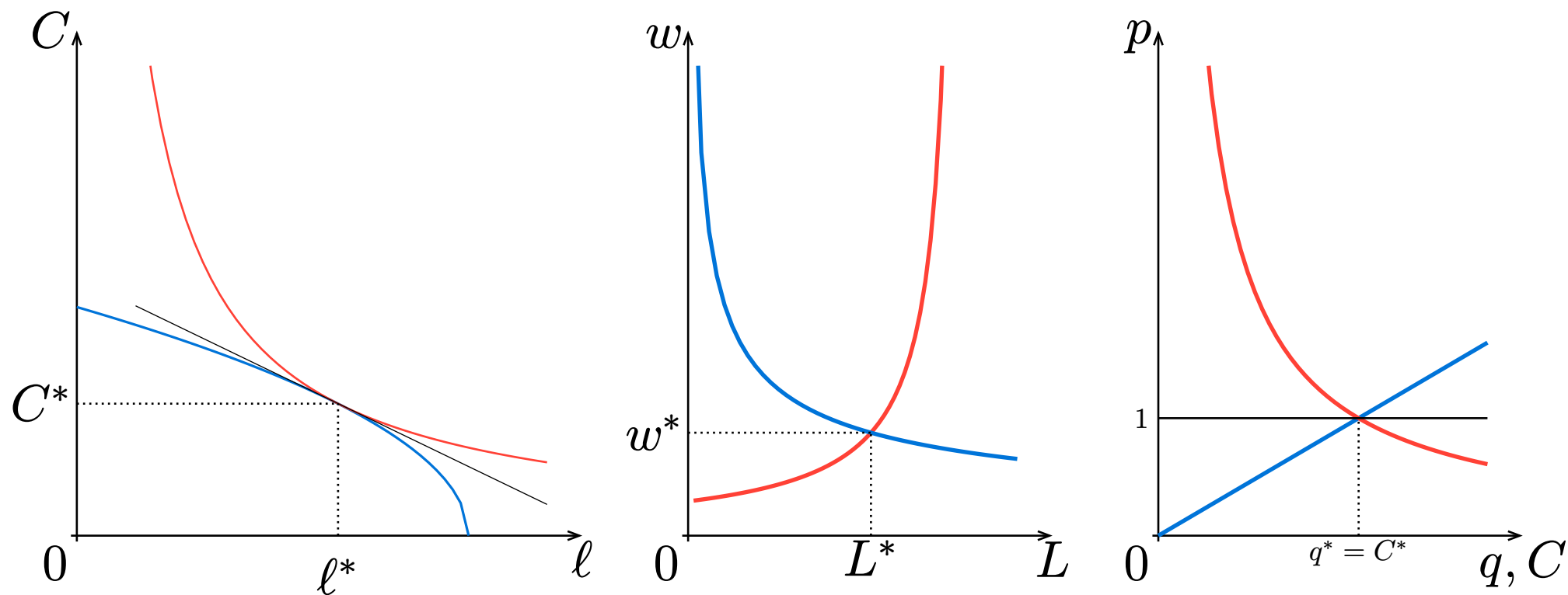
$$p^* C^* = w^* (T - \ell^*) + \pi^*$$

$$q^* = f(L^*)$$

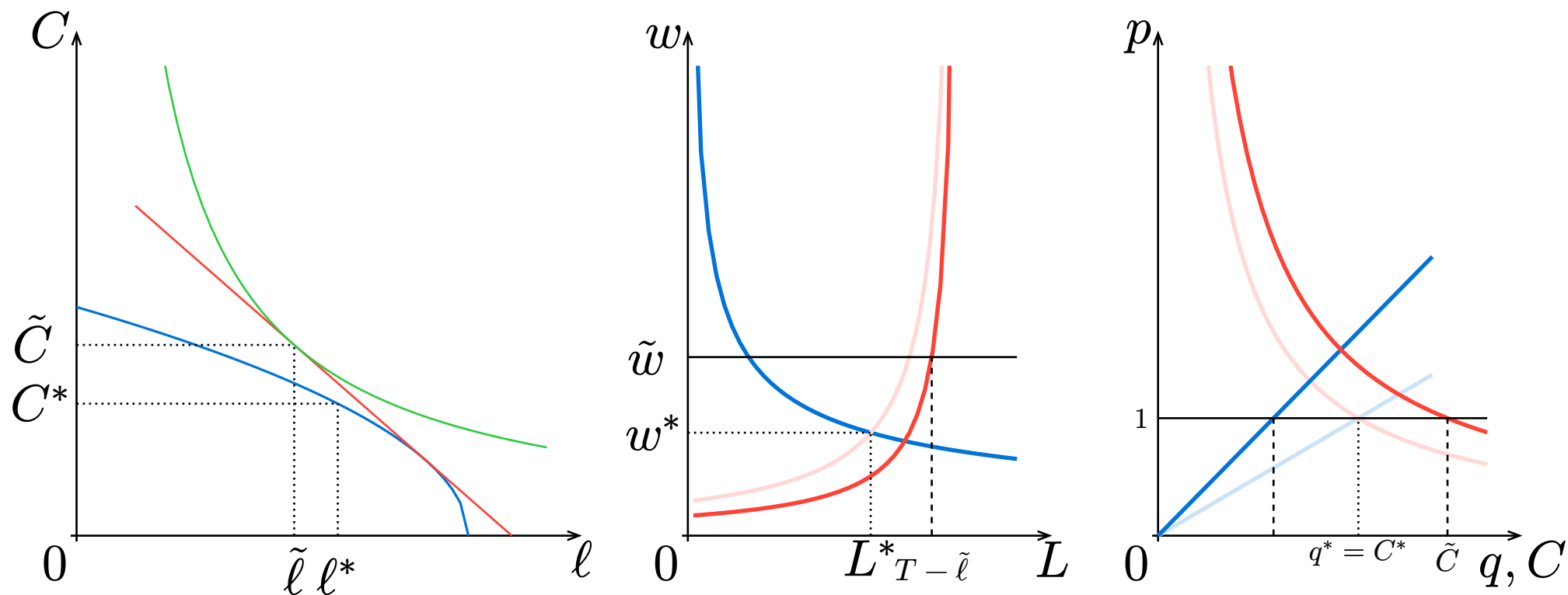
$$C^* = q^*$$

$$\ell^* = T - L^*$$

Explanation of the equilibrium condition



Explanation of the equilibrium condition



Efficiency of the equilibrium

Efficiency

Recall an informal definition of Pareto efficiency:

An allocation is Pareto efficient if there are no other **feasible** allocations at which everyone is not worse off and at least someone is **strictly** better off.

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Pareto efficiency in RCE

An allocation $(\bar{C}, \bar{\ell}, \bar{L})$ is Pareto-efficient if there are no other allocations (C, ℓ, L) such that:

$$L = T - \ell,$$

$$C = f(L),$$

and $U(C, \ell) > U(\bar{C}, \bar{\ell})$.

Social planner

How to practically find Pareto-efficient allocations?

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Social planner's problem

$$\begin{array}{ll} \max_{C, \ell, L} & \mathcal{W} \equiv U(C, \ell) & \text{definition of welfare} \\ \text{s.t.} & C = f(L) & \text{technological constraint 1} \\ & L = T - \ell & \text{technological constraint 2} \end{array}$$

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Important:

- Social planner chooses **real quantities only**
 - There are no prices in the problem

Efficiency conditions

$$\frac{\partial U(C^S, \ell^S)}{\partial C} = -\lambda$$

$$\frac{\partial U(C^S, \ell^S)}{\partial \ell} = -\mu$$

$$\lambda \frac{\partial f(L^S)}{\partial L} = \mu$$

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$$\frac{\frac{\partial U(C^S, \ell^S)}{\partial \ell}}{\frac{\partial U(C^S, \ell^S)}{\partial C}} = \frac{\partial f(L^S)}{\partial L} = \frac{\partial f(T - \ell^S)}{\partial L}$$

Efficiency in RCE

First Welfare Theorem for RCE

The general equilibrium allocation (C^*, ℓ^*, L^*) in RCE is efficient, that is

$$C^* = C^S,$$

$$\ell^* = \ell^S,$$

$$L^* = L^S,$$

where (C^S, ℓ^S, L^S) is the solution to the social planner problem.