

Monopoly

ECON 201 - Winter 2026

Week 5: February 4th

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Today's Agenda

1. Monopolist problem
2. Optimal pricing
3. Mark-up definition
4. Types of price discrimination

Monopolist problem

Recall

A firm chooses the level of output q and the composition of inputs z to maximize its profits π :

$$\max_{q,z} \quad \pi \equiv pq - c$$

$$\text{s.t.} \quad p = p^* \quad \text{perfect competition}$$

$$q = f(z) \quad \text{technological constraint}$$

$$c = w^* \cdot z \quad \text{cost of production}$$

- q is the output quantity
- z are the inputs (factors of production)
- f is the production function
- p^* is the market price for the produced good
- w^* are the market prices for the inputs

Monopolist's problem

A firm chooses the level of output q , the price p , and the composition of inputs z to maximize its profits π :

$$\max_{p, q, z} \quad \pi \equiv pq - c$$

$$\text{s.t.} \quad p = Q^{D^{-1}}(q) \quad (\text{inverse) consumer demand}$$

$$q = f(z) \quad \text{technological constraint}$$

$$c = w^* \cdot z \quad \text{cost of production}$$

Monopolist's problem

First-order condition:

$$p + \frac{\partial p}{\partial q} q = \frac{\partial c(q)}{\partial q}.$$

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Re-arrange to define a mark-up:

$$p \left[1 + \frac{\frac{\partial p}{p}}{\frac{\partial q}{q}} \right] = \frac{\partial c(q)}{\partial q} \Rightarrow \frac{p}{\partial c(q)/\partial q} = \frac{1}{1 + 1/\varepsilon(q)},$$

where $\varepsilon(q) = \frac{\partial q/q}{\partial p/p}$ (demand elasticity).

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where $\varepsilon(q) = \frac{\partial q/q}{\partial p/p}$ (demand elasticity).

Since usually $\frac{\partial p}{\partial q} \leq 0$,

$$p^M \geq \text{MC}(q^M) \Rightarrow \text{Monopolist has market power.}$$

Price discrimination

Willingness to pay

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They are willing to accept the offer as long as:

$$\underbrace{U(\bar{C}, I - P)}_{\text{Accept the offer}} \geq \underbrace{U(0, I)}_{\text{Refuse the offer}} .$$

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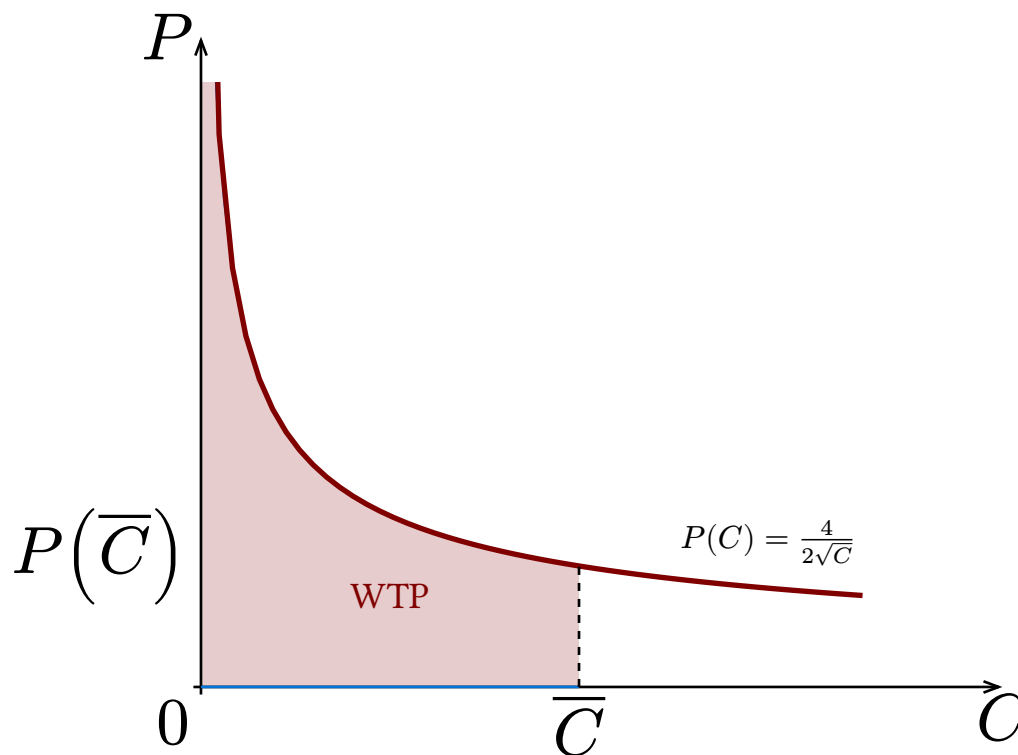
Willingness to pay for $\bar{C}$ is the maximum value of P that the consumer would pay to accept the offer:

$$U(0, I) = U(\bar{C}, I - W(\bar{C}))$$

Bridge between WTP and consumer surplus

Recall that the demand curve is derived from $\frac{\partial U(C)}{\partial C} = p$.

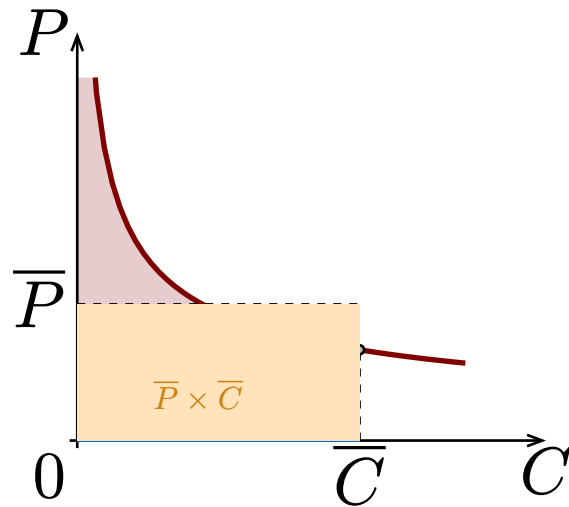
Recall also that $U(\bar{C}) = \int_0^{\bar{C}} \frac{\partial U(C)}{\partial C} dC$.



$$W(\bar{C}) = \int_0^{\bar{C}} P(C) dC = \int_0^{\bar{C}} \frac{\theta}{2\sqrt{C}} dC = \theta\sqrt{\bar{C}}$$

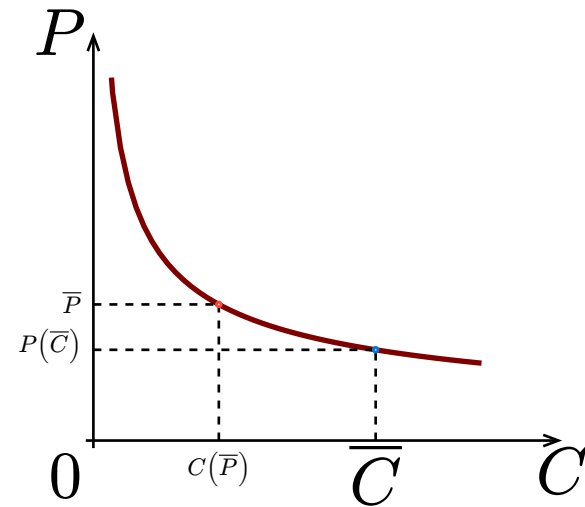
Difference between demand curve and WTP

Willingness to Pay



If either $\bar{C}$ or 0, would still pay $\bar{P}$ per unit.

Per-unit Pricing



If can choose any quantity, would only buy $C(\bar{P})$ for price $\bar{P}$.

Types of consumers

From now on, we will have two types of consumers:

- *High type*: Has high willingness to pay
- *Low type*: Has low willingness to pay

Formally, in case of $U_i(C, C_0) = \theta_i \sqrt{C} + C_0$:

$$\theta_1 > \theta_2.$$

Types of price discrimination

		<i>Information structure</i>	
		Full information	Incomplete information
<i>Pricing scheme</i>	Per-unit pricing	Simple monopoly + Third-degree PD	
	Lump-sum transfers	First-degree PD (personalized pricing)	Second-degree PD (screening/menus)

First-degree price discrimination

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First-degree price discrimination

- **Full information:**

Monopolist can verify with certainty the type of any consumer

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Monopolist offers combos of quantities and payments (T, q)

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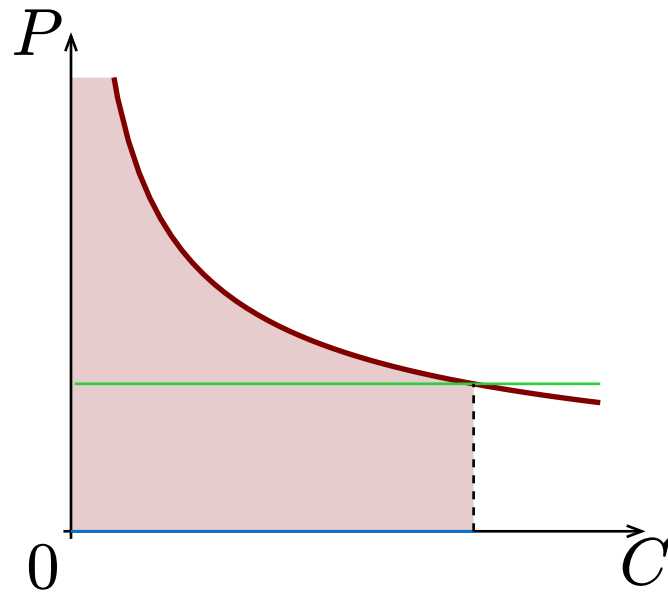
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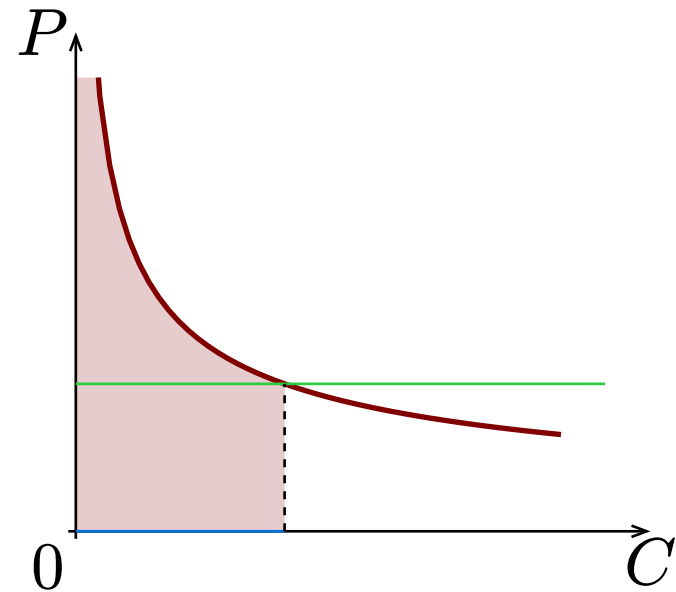
$$\begin{aligned} \max_{T_1, T_2, q_1, q_2} \quad & T_1 + T_2 - c(q_1 + q_2) \\ \text{s.t.} \quad & U_1(q_1, I_1 - T_1) \geq U_1(0, I_1) \\ & U_2(q_2, I_2 - T_2) \geq U_2(0, I_2) \end{aligned}$$

First-degree price discrimination

Type I (HWP)



Type II (LWP)



Two-part tariff

$$\begin{array}{ll} \max_{p,A,q} & pq + A - c(q) \\ \text{s.t.} & U(q, I - pq - A) \geq U(0, I) \end{array}$$

Second-degree price discrimination

		<i>Information structure</i>	
		Full information	Incomplete information
<i>Pricing scheme</i>	Per-unit pricing	Simple monopoly + Third-degree PD	
	Lump-sum transfers	First-degree PD (personalized pricing)	Second-degree PD (screening/menus)

Second-degree price discrimination

- **Incomplete information:**

Monopolist cannot verify the type of any consumer

Second-degree price discrimination

- **Incomplete information:**

Monopolist cannot verify the type of any consumer

- **Lump-sum transfers:**

Monopolist offers combos of quantities and payments (T, q)

Second-degree price discrimination

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Monopolist cannot verify the type of any consumer

- **Lump-sum transfers:**

Monopolist offers combos of quantities and payments (T, q)

$$\begin{aligned} \max_{T_1, T_2, q_1, q_2} \quad & T_1 + T_2 - c(q_1 + q_2) \\ \text{s.t.} \quad & U_1(q_1, I_1 - T_1) \geq U_1(0, I_1) \\ & U_2(q_2, I_2 - T_2) \geq U_2(0, I_2) \\ & U_1(q_1, I_1 - T_1) \geq U_1(q_2, I_1 - T_2) \\ & U_2(q_2, I_2 - T_2) \geq U_2(q_1, I_2 - T_1) \end{aligned}$$

2nd-degree PD: Price-setting

At the solution:

$$U_2(q_2, I_2 - T_2) = U_2(0, I_2)$$

$$U_1(q_1, I_1 - T_1) = U_1(q_2, I_1 - T_2)$$

This implies:

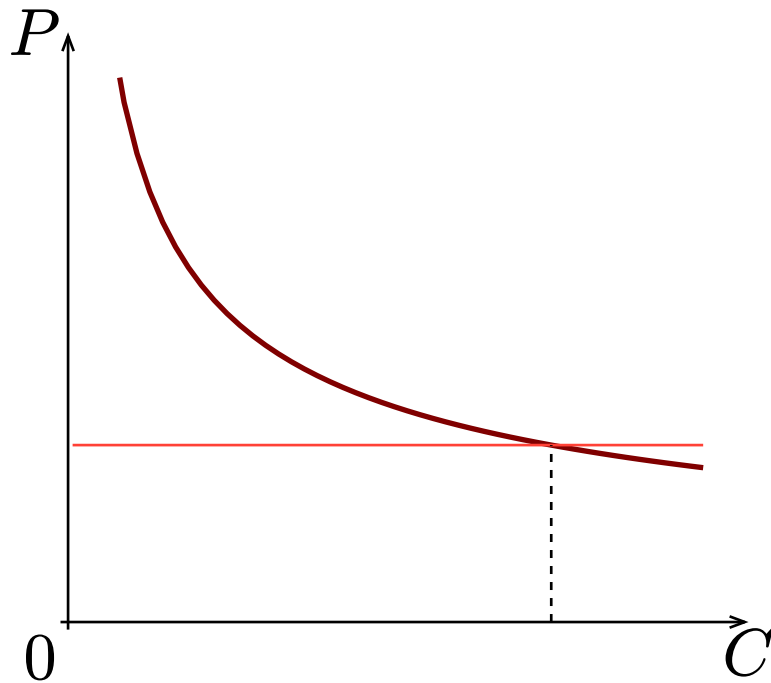
- Information rent of Type I:

$$U_1(q_1, I_1 - T_1) > U_1(0, I_1)$$

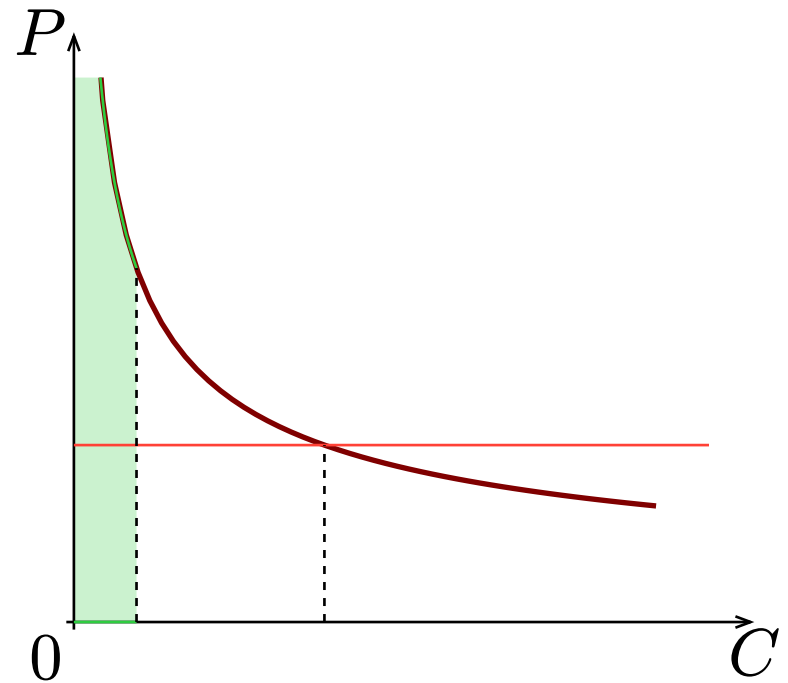
- q_2 is lower than in the 1st-degree PD

2nd-degree PD: Price-setting

Type I (HWP)

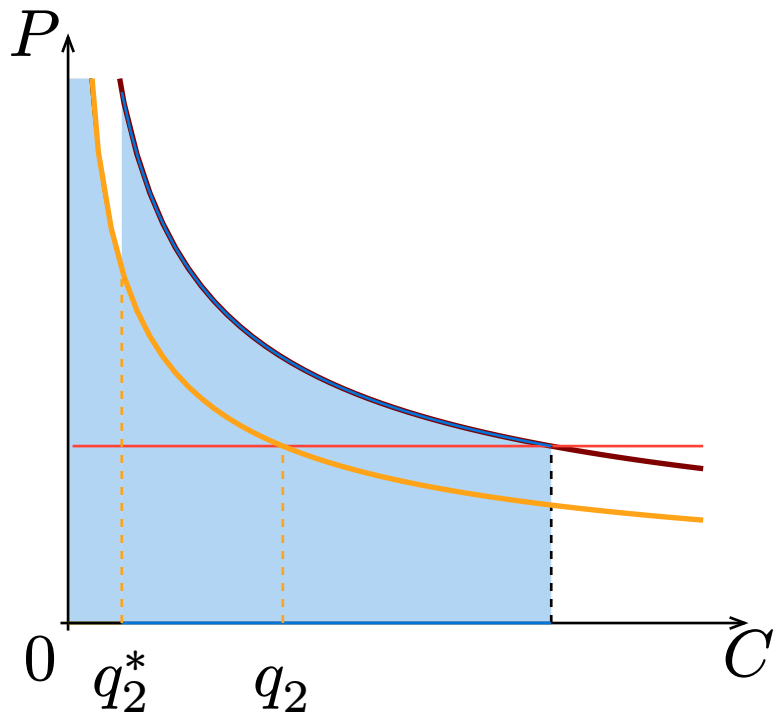


Type II (LWP)

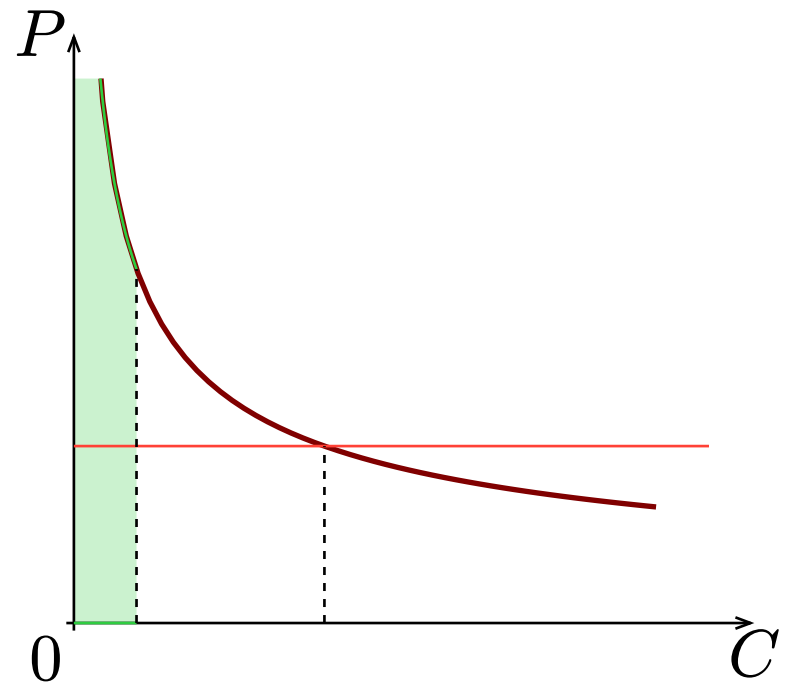


2nd-degree PD: Price-setting

Type I (HWP)



Type II (LWP)



Third-degree price discrimination

		<i>Information structure</i>	
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<i>Pricing scheme</i>	Per-unit pricing	Simple monopoly + Third-degree PD	
	Lump-sum transfers	First-degree PD (personalized pricing)	Second-degree PD (screening/menus)

Third-degree price discrimination

- **Incomplete information:**

Monopolist can verify the type of any consumer

Third-degree price discrimination

- **Incomplete information:**

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- **Per-unit pricing:**

Monopolist sets a per-unit price, consumers choose quantity

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$$\begin{aligned} \max_{p_1, p_2, q_1, q_2} \quad & p_1 q_1 + p_2 q_2 - c(q_1 + q_2) \\ \text{s.t.} \quad & p_1 = Q_1^{D^{-1}}(q_1) \\ & p_2 = Q_2^{D^{-1}}(q_2) \end{aligned}$$