

Oligopoly

ECON 201 - Winter 2026

Week 8: February 25th

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Today's Agenda

1. Game-theoretic set-up of an oligopolistic market
2. 2-firm Cournot equilibrium
3. n -firm Cournot equilibrium
4. Stackelberg equilibrium

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Goal: Describe the behavior of firms in strategic settings and compare different market structures.

Oligopolistic competition in quantities

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1. *Payoffs*: Profit $\pi_i(q_1, q_2)$ for firm i .

$$\pi_i(q_1, q_2) = \underbrace{p(q_1 + q_2)}_{\substack{\text{Inverse} \\ \text{demand function}}} q_i - c_i(q_i), \quad i \in \{1, 2\},$$

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Notice: Actions are continuous $\Rightarrow$ Payoff tables are not feasible.

Best-response functions

Each firm takes the decisions of the others as *given*.

Therefore, the best response function of firm 1 is a product of:

$$\begin{aligned} \max_{q_1} \quad & pq_1 - c_1(q_1) \\ \text{s.t.} \quad & p = q_D^{-1}(q_1 + q_2) \end{aligned}$$

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Therefore, we can solve for q_1 as a function of q_2 to get the best response function:

$$p(q_1 + \textcolor{red}{q}_2) + \frac{\partial p(q_1 + \textcolor{red}{q}_2)}{\partial q} q_1 = \frac{\partial c_1(q_1)}{\partial q_1}$$

Nash equilibrium

Both firms follow their best response functions simultaneously:

$$q_1^{\text{NE}} = \arg \max_{q_1} \{pq_1 - c_1(q_1) \mid p = q_D^{-1}(q_1 + q_2^{\text{NE}})\}$$

$$q_2^{\text{NE}} = \arg \max_{q_2} \{pq_2 - c_2(q_2) \mid p = q_D^{-1}(q_1^{\text{NE}} + q_2)\}$$

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Can be found by solving:

$$p(q_1 + q_2) + \frac{\partial p(q_1 + q_2)}{\partial q} q_1 = \frac{\partial c_1(q_1)}{\partial q_1}$$

$$p(q_2 + q_1) + \frac{\partial p(q_2 + q_1)}{\partial q} q_2 = \frac{\partial c_2(q_2)}{\partial q_2}$$

N -firm extension

Assume $q_D^{-1}(Q) = a - bQ$.

There are an arbitrary number N of firms on the market with a constant marginal cost c :

$$Q = \sum_{i=1}^N q_i$$

The system of FOCs becomes:

$$a - b \sum_{j=1}^N q_j - bq_i = c, \quad i = 1, 2, \dots, N$$

Due to the symmetric structure, the solution is symmetric:

$$q_i^{\text{NE}} = q_j^{\text{NE}} \quad \forall i, j = 1, 2, \dots, N.$$

N -firm extension

Nash equilibrium in the N -firm Cournot model

Under linear inverse demand $q_D^{-1}(Q) = a - bQ$ and constant identical marginal costs c , the Nash equilibrium in the N -firm Cournot model is given by:

$$q_i^{\text{NE}} = \frac{a - c}{b(N + 1)}, \quad i = 1, 2, \dots, N.$$

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Thus, the price and aggregate quantity are given by:

$$p^{\text{NE}} = a - b \sum_{i=1}^N q_i^{\text{NE}} = a - bN \frac{a - c}{b(N + 1)} = \frac{a + Nc}{N + 1}$$

$$Q^{\text{NE}} = \sum_{i=1}^N q_i^{\text{NE}} = N \frac{a - c}{b(N + 1)} = \frac{N}{N + 1} \frac{a - c}{b}$$

Comparing market structures

	Perfect competition	Cournot (N firms)	Monopoly (<i>per-unit pricing, no discrimination</i>)
Price			
Quantity			
Firm profit			
Consumer surplus			
Deadweight loss			

Under linear inverse demand $p = a - bQ$ and constant marginal cost c .

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Price	$p^* = c$	$p^* = \frac{a + Nc}{N + 1}$	$p^* = \frac{a + c}{2}$
Quantity	$Q^* = \frac{a - c}{b}$	$Q^* = \frac{N}{N + 1} \cdot \frac{a - c}{b}$	$Q^* = \frac{a - c}{2b}$
Firm profit	$\pi_i = 0$	$\pi_i = \frac{(a - c)^2}{b(N + 1)^2}$	$\pi = \frac{(a - c)^2}{4b}$
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Deadweight loss	$DWL = 0$	$DWL = \frac{(a - c)^2}{2b(N + 1)^2}$	$DWL = \frac{(a - c)^2}{8b}$

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- We will need the sequential game set-up to analyze this setting.

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Firm 2's best response

As in simple games, Firm 2 devises a strategy for any possible decision of Firm 1.

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Therefore, Firm 2's best response is the same as in the simultaneous set-up:

$$q_2(q_1) = \frac{a - c - bq_1}{2b}.$$

Firm 1's decision

Firm 1 takes advantage of moving first as it knows how Firm 2 will respond:

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First-order condition:

$$p + q_1 \frac{\partial p}{\partial q} \left[1 + \frac{\partial q_2}{\partial q_1} \right] = \frac{\partial c_1(q_1)}{\partial q_1}$$

Stackelberg equilibrium

Subgame-Perfect Nash Equilibrium in Stackelberg model

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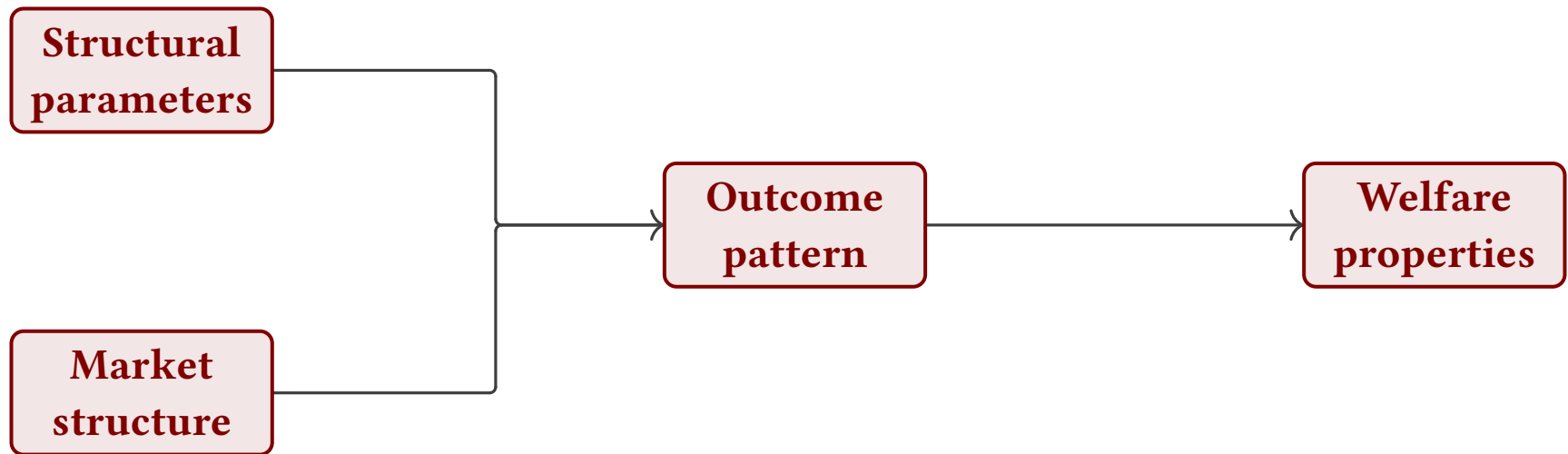
$$q_1^* = \frac{a - c}{2b} \quad q_2(q_1) = \frac{a - c}{2b} - \frac{1}{2}q_1$$

Thus, the equilibrium price and quantity are:

$$p^* = a - b \frac{3(a - c)}{4b} = \frac{a + 3c}{4}$$

$$Q^* = \frac{3}{4} \frac{a - c}{b}$$

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