

Externalities

ECON 201 - Winter 2026

Week 9: March 4th

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Today's Agenda

1. An economy with production externalities
2. Ways to address externalities:
 1. Private market solution: Coase Theorem
 2. Policy solutions: Pigouvian taxes and subsidies

Production externalities

Motivation

- So far, we looked at the interaction of firms who compete on the same market

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- There are cases when firms' decisions affect each other when they operate in unrelated markets

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- There are cases when firms' decisions affect each other when they operate in unrelated markets
 - ▶ A pharmaceutical company discovers a new technology to develop a vaccine that other companies can use to produce their own vaccines
 - ▶ A mining company opens a new mine that ruins the nature tourism industry in the neighboring areas
 - ▶ A big importer lobbies for a reduction in tariffs that all importers in the country are facing

Building blocks

- Two firms:
 - Operate on unrelated markets
 - Their markets are perfectly competitive
 - The production choices of one firm affect the production possibilities of the other firm

For the lecture: Assume a negative externality:

$$\frac{\partial f_2(L_2; v)}{\partial v} \frac{\partial g(q_1)}{\partial q_1} < 0.$$

The case of a positive externality is symmetric.

Firm 1

- $f_1(L_1) = \sqrt{L_1}$

$$\begin{aligned} \max_{q_1, L_1} \quad & p_1 q_1 - wL_1 \\ \text{s.t.} \quad & q_1 = f_1(L_1) \end{aligned}$$

Firm 1

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The FOCs:

$$\begin{aligned} p_1 &= -\lambda_1 \\ -w &= \lambda_1 \frac{\partial f_1(L_1)}{\partial L_1} \end{aligned}$$

Firm 2

- $f_2(L_2; v) = \frac{1}{1+v} \sqrt{L_2}$

$$\begin{aligned} \max_{q_2, L_2} \quad & p_2 q_2 - w L_2 \\ \text{s.t.} \quad & q_2 = f(L_2; v) \end{aligned}$$

Firm 2

- $f_2(L_2; v) = \frac{1}{1+v} \sqrt{L_2}$

$$\begin{aligned} \max_{q_2, L_2} \quad & p_2 q_2 - w L_2 \\ \text{s.t.} \quad & q_2 = f(L_2; v) \end{aligned}$$

- Pollution technology:

$$v = g(q_1) = q_1$$

Efficiency

Social planner's problem

$$\begin{aligned} \max_{C_1, C_2, v, L_1, L_2, \ell} \quad & U(C_1, C_2, \ell) \\ \text{s.t.} \quad & C_1 = f_1(L_1) \\ & C_2 = f_2(L_2; v) \\ & v = g(C_1) \\ & L_1 + L_2 + \ell = T \end{aligned}$$

Efficient allocation

$$\frac{\partial U(C_1, C_2, \ell)}{\partial C_1} = -\lambda_1 + \lambda_3 \frac{\partial g(C_1)}{\partial C_1}$$

$$\frac{\partial U(C_1, C_2, \ell)}{\partial C_2} = -\lambda_2$$

$$\frac{\partial U(C_1, C_2, \ell)}{\partial \ell} = -\lambda_4$$

$$\lambda_2 \frac{\partial f_2(L_2; v)}{\partial v} = \lambda_3$$

$$\lambda_1 \frac{\partial f_1(L_1)}{\partial L_1} = \lambda_4$$

$$\lambda_2 \frac{\partial f_2(L_2; v)}{\partial L_2} = \lambda_4$$

Social planner's trade-off

$$\frac{\lambda_1}{\lambda_2} \frac{\partial f_1(L_1)/\partial L_1}{\partial f_2(L_2;v)/\partial L_2} = 1 \Rightarrow \lambda_1 = - \frac{\frac{\partial U(C_1, C_2, \ell)}{\partial C_2} \frac{\partial f_2(L_2;v)}{\partial L_2}}{\partial f_1(L_1)/\partial L_1}$$

$$\lambda_4 = - \frac{\partial U(C_1, C_2, \ell)}{\partial C_2} \frac{\partial f_2(L_2;v)}{\partial L_2}$$

$$\frac{\partial U(C_1, C_2, \ell)}{\partial C_1} - \frac{\frac{\partial U(C_1, C_2, \ell)}{\partial C_2} \frac{\partial f_2(L_2;v)}{\partial L_2}}{\partial f_1(L_1)/\partial L_1} = - \frac{\partial U(C_1, C_2, \ell)}{\partial C_2} \frac{\partial f_2(L_2;v)}{\partial v} \frac{\partial g(C_1)}{\partial C_1}$$

Thus, the optimality condition:

$$\frac{\partial U(C_1, C_2, \ell)/\partial C_1}{\partial U(C_1, C_2, \ell)/\partial C_2} - \frac{\partial f_2(L_2;v)/\partial L_2}{\partial f_1(L_1)/\partial L_1} = - \frac{\partial f_2(L_2;v)}{\partial v} \frac{\partial g(C_1)}{\partial C_1}$$

Social planner's trade-off

Consider a marginal reallocation of labor from L_2 to L_1 :

$$\underbrace{\frac{\partial U(C_1, C_2, \ell)}{\partial C_1} \frac{\partial f_1(L_1)}{\partial L_1}}_{\text{welfare benefit from increasing } L_1} = \frac{\partial U(C_1, C_2, \ell)}{\partial C_2} \left[\underbrace{\frac{\partial f_2(L_2; v)}{\partial L_2}}_{\text{direct effect of reducing } L_2} - \underbrace{\frac{\partial f_2(L_2; v)}{\partial v} \frac{\partial g(C_1)}{\partial C_1} \frac{\partial f_1(L_1)}{\partial L_1}}_{\text{indirect effect of increasing } L_1 \text{ through increasing pollution}} \right]$$

$\underbrace{\hspace{15em}}_{\text{welfare cost of re-allocation from } L_2 \text{ to } L_1}$

Proving that $v^{\text{SP}} < v^*$

Overview

Strategy: Determine how both sides of the optimality condition change with respect to $\frac{L_1}{L_2}$.

Preview: The left-hand side decreases with $\frac{L_1}{L_2}$ and the right-hand side is always positive $\Rightarrow$ Left-hand side crosses 0 to the right of its crossing with the right-hand side.

Keep in mind: All of the variables in the equilibrium and in the efficient allocation satisfy:

$$C_1^* = f_1(L_1^*)$$

$$\Downarrow$$

$$v^* = g(C_1^*) = g(f_1(L_1^*))$$

$$\Downarrow$$

$$C_2^* = f_2(L_2^*; v^*) = f_2(L_2^*; g(f_1(L_1^*)))$$

Assumptions

First, let's collect all the assumption we are making:

- Decreasing marginal utility of consumption:

$$\frac{\partial^2 U(C_1, C_2, \ell)}{\partial^2 C_1} < 0 \quad \frac{\partial^2 U(C_1, C_2, \ell)}{\partial^2 C_2} < 0$$

- Decreasing marginal product of labor:

$$\frac{\partial^2 f_1(L_1)}{\partial^2 L_1} < 0 \quad \frac{\partial^2 f_2(L_2; v)}{\partial^2 L_2} < 0$$

- Decreasing marginal pollution per unit of output:

$$\frac{\partial^2 g(C_1)}{\partial^2 C_1} < 0$$

Proving that $v^{\text{SP}} < v^*$

$$\frac{C_1}{C_2} = \frac{f_1(L_1)}{f_2(L_2; v)} \rightarrow \text{increases with } \frac{L_1}{L_2}$$

$$\frac{\partial U / \partial C_1}{\partial U / \partial C_2} \text{ decreases with } \frac{C_1}{C_2} \Rightarrow \text{decreases with } \frac{L_1}{L_2}$$

$$\frac{\partial f_2(L_2; v) / \partial L_2}{\partial f_1(L_1) / \partial L_1} \text{ increases with } \frac{L_1}{L_2}$$

$$\frac{\partial f_2(L_2; v)}{\partial v} \frac{\partial g(C_1)}{\partial C_1} < 0 \Rightarrow$$

$$-\frac{\partial f_2(L_2; v)}{\partial v} \frac{\partial g(C_1)}{\partial C_1} > 0$$

Bringing together

$$\underbrace{\underbrace{\frac{\partial U(C_1, C_2, \ell)/\partial C_1}{\partial U(C_1, C_2, \ell)/\partial C_2}}_{\text{decreases with } \frac{L_1}{L_2}} - \underbrace{\frac{\partial f_2(L_2; v)/\partial L_2}{\partial f_1(L_1)/\partial L_1}}_{\text{increases with } \frac{L_1}{L_2}}}_{\text{decreases with } \frac{L_1}{L_2}} = - \underbrace{\underbrace{\frac{\partial f_2(L_2; v)}{\partial v} \frac{\partial g(C_1)}{\partial C_1}}_{<0}}_{>0}$$

$\Downarrow$

$$\frac{L_1^{\text{SP}}}{L_2^{\text{SP}}} < \frac{L_1^*}{L_2^*}$$

$\Downarrow$

$$v^{\text{SP}} < v^*$$

Example

- Utility function

$$U(C_1, C_2, \ell) = C_1^{\frac{1}{3}} C_2^{\frac{1}{3}} \ell^{\frac{1}{3}}$$

- Firm 1's technology

$$f_1(L_1) = L_1^{\frac{1}{2}}$$

- Firm 2's technology

$$f_2(L_2; v) = \frac{1}{1+v} L_2^{\frac{1}{2}}$$

- Pollution technology

$$v = q_1$$

Solving individual problems of agents

Consumer's optimality condition:

$$\frac{C_1^{-\frac{2}{3}} C_2^{\frac{1}{3}} \ell^{\frac{1}{3}}}{C_1^{\frac{1}{3}} C_2^{-\frac{2}{3}} \ell^{\frac{1}{3}}} = \frac{p_1}{p_2} \Leftrightarrow \frac{C_2}{C_1} = \frac{p_1}{p_2}$$

Firm 1's optimality condition:

$$\frac{1}{2} \frac{1}{\sqrt{L_1}} = \frac{w}{p_1}$$

Firm 2's optimality condition:

$$\frac{1}{2} \frac{1}{1+v} \frac{1}{\sqrt{L_2}} = \frac{w}{p_2}$$

Equilibrium

Combining the optimality conditions:

$$\frac{C_2}{C_1} = \frac{1}{1+v} \frac{\sqrt{L_1}}{\sqrt{L_2}}$$

Using the market clearing in the output market:

$$\frac{\frac{1}{1+v} \sqrt{L_2}}{\sqrt{L_1}} = \frac{1}{1+v} \frac{\sqrt{L_1}}{\sqrt{L_2}} \Rightarrow$$
$$L_1^* = L_2^*$$

In equilibrium, labor is split equally between the two firms.

Efficient allocation

The optimality condition:

$$\frac{C_2}{C_1} - \frac{1}{1+v} \frac{\sqrt{L_1}}{\sqrt{L_2}} = \frac{1}{(1+v)^2} \sqrt{L_2}$$

Using the technology constraints:

$$\frac{\frac{1}{1+v} \sqrt{L_2}}{\sqrt{L_1}} - \frac{1}{1+v} \frac{\sqrt{L_1}}{\sqrt{L_2}} = \frac{1}{(1+v)^2} \sqrt{L_2}$$

After algebraic transformations:

$$L_2 = (1 + \sqrt{L_1}) L_1$$

Since $(1 + \sqrt{L_1}) L_1 > L_1$, in the efficient allocation, relatively more labor is allocated to firm 2, compared to the market equilibrium.

The Coase Theorem

Perfect property rights

- What if the affected firm had enforceable property rights over v ?

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- What if the affected firm had enforceable property rights over v ?
- It could go to court and ask for compensation from the other firm for the damage.

Perfect property rights

- What if the affected firm had enforceable property rights over v ?
- It could go to court and ask for compensation from the other firm for the damage.
- Or, it could directly sell permits to the other firm.

Firm 1

$$\begin{aligned} \max_{q_1, L_1, v} \quad & p_1 q_1 - w L_1 - p_v v \\ \text{s.t.} \quad & q_1 = f_1(L_1) \\ & v = g(q_1) \end{aligned}$$

Firm 1

$$\begin{aligned} \max_{q_1, L_1, v} \quad & p_1 q_1 - w L_1 - p_v v \\ \text{s.t.} \quad & q_1 = f_1(L_1) \\ & v = g(q_1) \end{aligned}$$

The FOCs:

$$\begin{aligned} p_1 &= -\lambda_1 + \lambda_2 \frac{\partial g(q_1)}{\partial q_1} \\ -w &= \lambda_1 \frac{\partial f_1(L_1)}{\partial L_1} \\ p_v &= \lambda_2 \end{aligned}$$

Firm 2

- $f_2(L_2; v) = \frac{1}{1+v} \sqrt{L_2}$

$$\max_{q_2, L_2, v} \quad p_2 q_2 - w L_2 + p_v v$$

$$\text{s.t.} \quad q_2 = f_2(L_2; v)$$

Firm 2

$$\bullet \quad f_2(L_2; v) = \frac{1}{1+v} \sqrt{L_2}$$

$$\begin{aligned} \max_{q_2, L_2, v} \quad & p_2 q_2 - w L_2 + p_v v \\ \text{s.t.} \quad & q_2 = f_2(L_2; v) \end{aligned}$$

The FOCs:

$$p_2 = \mu_1$$

$$w = \mu_1 \frac{\partial f_2(L_2; v)}{\partial L_2}$$

$$p_v = -\mu_1 \frac{\partial f_2(L_2; v)}{\partial v}$$

General equilibrium

Equilibrium definition

An equilibrium in the Economy with Perfect Property Rights consists of levels of consumption C_1^* , C_2^* , levels of production q_1^* , q_2^* , level of leisure ℓ^* , levels of labor inputs L_1^* , L_2^* , level of environmental damage v^* , prices p_1^* , p_2^* , wages w_1^* , w_2^* such that

- C_1^* , C_2^* , and ℓ^* solve the consumer's problem given p_1^* , p_2^* , w_1^* , w_2^* , π_1^* , π_2^* ;
- q_1^* , q_2^* , L_1^* , L_2^* , and v^* solve the firms' problems given p_1^* , p_2^* , w_1^* , w_2^* , p_v^* ;
- the output market, the labor market, and the market for environmental damage permits clear.

Market clearing

Output market

$$C_1^* = q_1^*, \quad C_2^* = q_2^*$$

Labor market

$$L_1^* = s_1^*(T - \ell^*), \quad L_2^* = s_2^*(T - \ell^*)$$

Environmental damage market

The level of environmental damage that firm 2 is willing to accept matches the level that firm 1 wants.

$$v^* = g(q_1^*)$$

Equilibrium conditions

$$\frac{w}{p_1 + p_v \frac{\partial g(q_1^*)}{\partial q_1}} = \frac{\partial f_1(L_1)}{\partial L_1}$$

$$\frac{w}{p_2} = \frac{\partial f_2(L_2; v)}{\partial L_2}$$

$$\frac{p_v}{p_2} = \frac{\partial f_2(L_2; v)}{\partial v}$$

$$\frac{p_1}{p_2} = \frac{\partial U(C_1^*, C_2^*, \ell^*) / \partial C_1}{\partial U(C_1^*, C_2^*, \ell^*) / \partial C_2}$$

Determination of environmental damage

$$\frac{p_1}{p_2} + \frac{p_v}{p_2} \frac{\partial g(q_1^*)}{\partial q_1} = \frac{\partial f_2(L_2; v) / \partial L_2}{\partial f_1(L_1) / \partial L_1}$$

Coase theorem

Efficiency in the Economy with Perfect Property Rights

If property rights are well-defined and enforceable at zero cost, then the general equilibrium allocation is efficient.

Pigouvian taxation

Firm 1

$$\begin{aligned} \max_{q_1, L_1} \quad & (p_1 - t)q_1 - wL_1 \\ \text{s.t.} \quad & q_1 = f_1(L_1) \end{aligned}$$

Firm 1

$$\begin{aligned} \max_{q_1, L_1} \quad & (p_1 - t)q_1 - wL_1 \\ \text{s.t.} \quad & q_1 = f_1(L_1) \end{aligned}$$

The FOCs:

$$\begin{aligned} p_1 - t &= -\lambda_1 \\ -w &= \lambda_1 \frac{\partial f_1(L_1)}{\partial L_1} \end{aligned}$$

Firm 2

- $f_2(L_2; v) = \frac{1}{1+v} \sqrt{L_2}$

$$\max_{q_2, L_2} p_2 q_2 - w L_2$$

$$\text{s.t. } q_2 = f_2(L_2; v)$$

$$p_2 = \mu_1$$

$$-w = \mu_1 \frac{\partial f_2(L_2; v)}{\partial L_2}$$

General equilibrium

Equilibrium definition

An equilibrium in the Economy with Production Taxes consists of levels of consumption C_1^* , C_2^* , levels of production q_1^* , q_2^* , level of leisure ℓ^* , levels of labor inputs L_1^* , L_2^* , level of environmental damage v^* , prices p_1^* , p_2^* , wages w_1^* , w_2^* such that

- C_1^* , C_2^* , and ℓ^* solve the consumer's problem given p_1^* , p_2^* , w_1^* , w_2^* , π_1^* , π_2^* ;
- q_1^* , q_2^* , L_1^* , and L_2^* solve the firms' problems given p_1^* , p_2^* , w_1^* , w_2^* , and production tax t ;
- the output market and the labor market clear.

Market clearing

Output markets

$$C_1^* = q_1^*, \quad C_2^* = q_2^*$$

Labor markets

$$L_1^* = s_1^*(T - \ell^*), \quad L_2^* = s_2^*(T - \ell^*)$$

Equilibrium conditions

$$\frac{w}{p_1 - t} = \frac{\partial f_1(L_1)}{\partial L_1}$$

$$\frac{w}{p_2} = \frac{\partial f_2(L_2; v)}{\partial L_2}$$

$$\frac{p_1}{p_2} = \frac{\partial U(C_1^*, C_2^*, \ell^*) / \partial C_1}{\partial U(C_1^*, C_2^*, \ell^*) / \partial C_2}$$

Determination of environmental damage

$$\frac{p_1}{p_2} - \frac{t}{p_2} = \frac{\partial f_2(L_2; v) / \partial L_2}{\partial f_1(L_1) / \partial L_1}$$

Determination of environmental damage

$$\frac{p_1}{p_2} - \frac{t}{p_2} = \frac{\partial f_2(L_2; v) / \partial L_2}{\partial f_1(L_1) / \partial L_1}$$

Pigouvian tax

$$\begin{aligned} t^* &= -p_2 \frac{\partial f_2(L_2^*; v)}{\partial v} \frac{\partial g(q_1^*)}{\partial q_1} \\ &= -\frac{\partial U(C_1^*, C_2^*, \ell^*)}{\partial C_2} \frac{\partial f_2(L_2^*; v)}{\partial v} \frac{\partial g(q_1^*)}{\partial q_1} \end{aligned}$$

General equilibrium with Pigouvian taxes

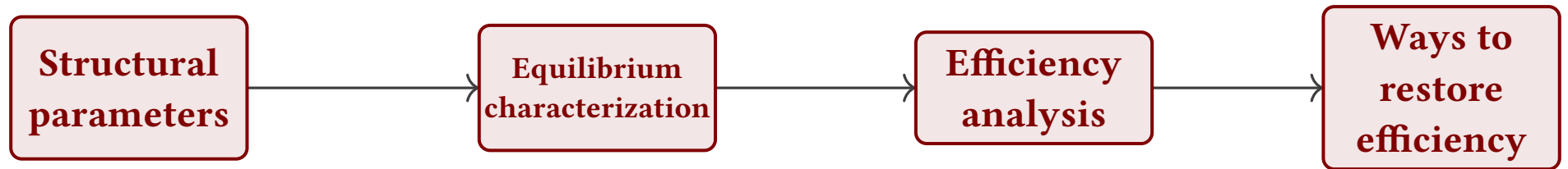
Efficiency in the Economy with Pigouvian taxes

If the tax t is set at

$$t^* = - \frac{\partial U(C_1^*, C_2^*, \ell^*)}{\partial C_2} \frac{\partial f_2(L_2^*; v)}{\partial v} \frac{\partial g(q_1^*)}{\partial q_1},$$

then the general equilibrium allocation in the economy with taxes is efficient.

Overview: Analysis pipeline



Overview: Analysis pipeline

